Complex Analysis (Sem I and III)

1. Let $z, w \in \mathbb{C}$. Then $|| z | - | w || \leq$ ....
   A. $| z | - | w |$.
   B. $| z - w |$.
   C. $z - w$.
   D. $z + w$.

2. The radius of convergence of the power series $\sum_{n=0}^{\infty}(8+6i)^n z^n$ is ....
   A. 10.
   B. $\frac{1}{10}$.
   C. 100.
   D. 20.

3. The length of the curve $\gamma(t) = 3e^{it}$, $t \in [0, 2\pi]$, is ....
   A. 2π
   B. π
   C. $\frac{\pi}{2}$
   D. 6π

4. $\int_\gamma \frac{z^2+z+2}{z} \, dz =$ .... ,where γ is the circle $|z| = 5$
   A. $i\pi$
   B. $4i\pi$
   C. $10i\pi$
   D. zero

5. Let f be analytic in open disk $B(a; R) \subset \mathbb{C}$ and suppose $|f(z)| \leq M$ for all z in $B(a; R)$. Then $|f^{(2)}(a)| \leq$ ....
   A. $\frac{Mn!}{R^n}$
   B. $\frac{M}{R^n}$
   C. $\frac{6M}{R^3}$
   D. $\frac{2M}{R^2}$

6. $\int_\gamma \frac{\sin z + z^2}{z-4} \, dz =$ .... , where γ is the circle $|z| = \frac{3}{2}$.
   A. $2\pi i$

B. $4\pi i$

C. zero

D. 16π

7. Let G be a domain in $\mathbb{C}$ and suppose that f is a non constant analytic function on G. Then for any open set U in G , ...

A. $f(U)$ is closed.

B. $f(U)$ is open.

C. $f(U)$ is neither open nor closed.

D. $f(U)$ is both open and closed.

8. Let $z = z_0$ be an isolated singularity of f and let $f(z) = \sum_{n=-\infty}^{\infty} c_n(z - z_0)^n$ be its Laurent series expansion in $ann(z_0; 0, R)$. If $c_n = 0$ for $n \leq -1$ then $z = z_0$ ....

A. is a pole of order n.

B. is a removable singularity.

C. is a non isolated singularity.

D. is an essential singularity.

9. If f is analytic in $0 < |z - \alpha| < R$ and f has a pole of order 8 at $z = \alpha$ so that $f(z) = \frac{g(z)}{(z-\alpha)^8}$, where g is analytic in $|z - \alpha| < R$ then the residue of f at $z = \alpha$ is ....

A. $\dfrac{g^{(8)}(\alpha)}{7!}$

B. $\dfrac{g^{(5)}(\alpha)}{4!}$

C. $\dfrac{g^{(8)}(\alpha)}{8!}$

D. $\dfrac{g^{(7)}(\alpha)}{7!}$

10. Let $f(z) = \dfrac{z^2}{(z-3)(z+2)^2}$. Then the residue of f at $z = 3$ is ....

A. $\frac{4}{16}$

B. $\frac{9}{24}$

C. $\frac{9}{16}$

D. $\frac{9}{25}$

11. Let $f(z) = z^6 - 5z^4 + z^3 - 2z$ then $f(z)$ has .... zeros, counting multiplicities , inside the circle $|z| = 1$.

A. 3

B. 6

C. 4

D. 2

12. Let $z = x + iy$ and $w = u + iv$. The image of straight line $x = \frac{1}{4}$ in the complex plane under the transformation $w = \frac{1}{z}$ is ....,

A. $u^2 + v^2 - 4u = 0$

B. $u^2 + v^2 - 4u = 4$

C. $u^2 + v^2 - 4v = 0$

D. none of these

13. Let $S(z) = \dfrac{4z+5}{6z+7}$ be a mobius transformation. Then $S^{-1}(z) =$ ....

A. $s(z) = \dfrac{7z-5}{-6z+4}$

B. $s(z) = \dfrac{7z-5}{6z-4}$

C. $s(z) = \dfrac{7z+5}{-6z+4}$

D. $s(z) = \dfrac{6z-5}{4z-7}$

Differential Geometry

1. The value of t for which the vector $(3, 1, t)$ is parallel to the plane $2x + 4y + 5z = 12$ is

   (A) -4

   (B) -3

   (C) -2

   (D) -1

2. The equation of hyperplane passing through points $p = (1, 2, 1)$, $q = (-2, -1, 3)$ and $r = (2, -3, -1)$

   (A) $8x + 2y + 9z = 13$

   (B) $8x + 2y - 9z = 13$

   (C) $8x - 2y + 9z = 13$

   (D) $8x - 2y - 9z = 13$

3. Let S_1 be unit sphere in $\mathbb{R}^3$ center at origin and S_2 be another sphere in $\mathbb{R}^3$ of radius 5 center at origin. Let γ_1 and γ_2 be two diametric circles on S_1 and S_2 respectively, and $\kappa_i,\ \tau_i$ are curvatures and torsions of γ_i respectively, for $i = 1,\ 2$. Then

   (A) $\kappa_1 = \kappa_2$ and $\tau_1 < \tau_2$

   (B) $\kappa_1 = \kappa_2$ and $\tau_1 > \tau_2$

   (C) $\kappa_1 < \kappa_2$ and $\tau_1 = \tau_2$

   (D) $\kappa_1 > \kappa_2$ and $\tau_1 = \tau_2$.

4. The arc-length of one complete turn of the circular helix $\gamma(t) = (a\cos t,\ a\sin t,\ bt)$ for $a,\ b \in \mathbb{R}$

(A) $\pi\sqrt{a^2+b^2}$

(B) $4\pi\sqrt{a^2+b^2}$

(C) $2\pi\sqrt{a^2+b^2}$,

(D) $3\pi\sqrt{a^2+b^2}$.

5. For $\gamma : (0, 2\pi) \longrightarrow \mathbb{R}^3$, $\gamma(t) = \frac{1}{\sqrt{2}}(\int_t^{\pi/2} \sin s\ ds,\ t,\ \int_0^t \cos s\ ds)$. Curvature κ of γ at $(0,\frac{1}{\sqrt{2}},0)=\gamma(1)$ is

(A) $\frac{\pi}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{2\pi}{\sqrt{2}}$

(D) $2\sqrt{2}$.

6. If S with the parametrization X for open U and if $\alpha : [0,1] \longrightarrow S$ a regular parametrized curve. Then the tangent surface of α is

(A) $\mathbf{x}(t,v) = \alpha(t) + v\alpha'(t)$, for $(t,v) \in [0,1] \times I\!R$

(B) $\mathbf{x}(t,v) = v\alpha(t) + \alpha'(t)$, for $(t,v) \in [0,1] \times I\!R$

(C) $\mathbf{x}(t,v) = v\alpha(t) + v\alpha'(t)$, for $(t,v) \in [0,1] \times I\!R$

(D) $\mathbf{x}(t,v) = v(\alpha(t) + t\alpha'(t))$, for $(t,v) \in [0,1] \times I\!R$.

7. Let $X : U \subset I\!R^2 \longrightarrow I\!R^3$, for each $q \in U$, the differential $dX_q : I\!R^2 \longrightarrow I\!R^3$ is one to one is equivalent to say that

(A) $X_u \times X_v = 0$

(B) $X_u \times X_v \neq 0$

(C) two column vectors of the matrix dX_q to be linearly dependent

(D) the minor of order two of the matrix dX_q be zero.

8. First fundamental form I_q of a plane $P \subset \mathbb{R}^3$ passing through $q = (1, 0, 0)$ containing vectors $(1, 1, 0)$ and $(1, 0, 1)$ is

(A) $du^2 + 2dudv + dv^2$

(B) $du^2 + dv^2$

(C) $du^2 - dv^2$

(D) $du^2 - 2dudv + dv^2$

9. Let $S \subset \mathbb{R}^3$ be a regular surface and $X : U \subset \mathbb{R}^2 \longrightarrow S$ is its surface patch; if k_1, k_2 are principal curvatures of surface patch x at point p, then

(A) k_1, k_2 are real numbers

(B) there exist a tangent vector to x at p which is not a principal vector

(C) k_1, k_2 can not be real numbers

(D) k_1, k_2 are purely imaginary numbers

10. Let $p \in S$ and let $dN_p : T_p(S) \longrightarrow T_p(S)$ be a differential of the Gauss map where $T_p(S)$ denotes tangent space of surface S at point p. A point on a surface S is parabolic if

(A) $det(dN_p) = 0$

(B) $det(dN_p) > 0$

(C) $det(dN_p) < 0$

(D) $det(dN_p) = 0$ with $dN_p \neq 0$

Sem-I
PSMT104/PAMT104
DISCRETE MATHEMATICS
(Sample Questions)

1. If $56x + 72y = 40$, where x and y are integers. Then
   - A. There finitely many possibilities for x and y.
   - B. $x = -15 + 7t, y = 20 - 9t$, where $t \in \mathbb{Z}$.
   - C. $x = 20 + 9t, y = -15 - 7t$, where $t \in \mathbb{Z}$.
   - D. $x = 20 + 7t, y = -15 - 9t$, where $t \in \mathbb{Z}$.

2. The equation $x^3 + 6x^2 + 11x + 6 = 0$ has
   - A. Three non-real roots.
   - B. Three real roots.
   - C. Two non-real roots and one real root.
   - D. Two real roots and one non-real root.

3. Let f be a function from a set with $k + 1$ or more elements to a set with k elements. Then
   - A. f is not injective.
   - B. f is injective.
   - C. f is bijective.
   - D. f is injective but can not be surjective.

4. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are selected?
   - A. 6.
   - B. 7.
   - C. 9.
   - D. 8.

5. At a party, seven gentlemen check their hats. In how many ways can their hats be returned so that at least two of the gentlemen receive their own hats?
   - A. $7! - D_7$.
   - B. $7! - D_7 - 7 \times D_6$..
   - C. $7! - D_7 - D_6$.
   - D. $D_7 - D_6$.

6. If $r(m, n)$ denote the Ramsey number then which of the following is true.

A. $r(3,5) < 14$ and $r(4,4) < 18$.

B. $r(3,5) = 14$ and $r(4,4) < 18$.

C. $r(3,5) = 14$ and $r(4,4) = 18$.

D. $r(3,5) < 14$ and $r(4,4) = 18$.

7. Find the exponential generating function for the sequence $\{a_n\}$, where $a_n = n+1, n = 0, 1, 2, \ldots$.

   A. $(x+1)e^x$

   B. xe^x

   C. e^x

   D. $2xe^x$

8. Find the coefficient of x^{10} in the power series of $1/(1-x)^3$

   A. 64

   B. 65

   C. 67

   D. 66

9. Find the sequence with $f(x) = e^{3x} - 3e^{2x}$ as its exponential generating function.

   A. $a_n = 3^{n+1} - 3 \times 2^{n+1}$

   B. $a_n = 3^{n-1} - 3 \times 2^{n-1}$

   C. $a_n = 3^n - 3 \times 2^n$

   D. $a_n = 3^{n+1} + 3 \times 2^{n+1}$

10. Suppose a necklace can be made from beads of three colors: black, white, and red. How many different necklaces with 3 beads are there if we assume that only the cyclic group of order three (without any reflections) acts?

   A. 8

   B. 9

   C. 10

   D. 11

Sem-IV
PSMT401/PAMT401
FIELD THEORY
(Sample Questions)

1. The extension $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$ is:
   - A. not algebraic
   - B. not finite
   - C. algebraic
   - D. of degree four
2. Let $\omega \neq 1$ be a cube root of unity. Then, the degree of the extension $\mathbb{Q}(\omega)$ over $\mathbb{Q}$ is:
   - A. Four
   - B. Two
   - C. Three
   - D. One
3. The degree of the splitting field of $X^5 - 2$ over $\mathbb{Q}$ is:
   - A. Twenty
   - B. Ten
   - C. Five
   - D. One
4. Which of these constructions is possible using ruler and compass?
   - A. Trisecting angles
   - B. Doubling cubes
   - C. Equilateral triangles
   - D. Squaring circles
5. Consider the extension $L = \mathbb{Q}(\sqrt[3]{2})$. Then the extension $L/\mathbb{Q}$ is
   - A. Not algebraic
   - B. Not normal
   - C. Not finite
   - D. Not separable.
6. The degree of the extension $\mathbb{Q}(\sqrt[3]{17}, \sqrt{19})$ over $\mathbb{Q}$ is
   - A. Three
   - B. Two

C. Six

D. One

7. Let F be a field of characteristic zero. Which of the following statements is true?

   A. Every irreducible polynomial over F is separable.

   B. Every polynomial over F is separable.

   C. Every polynomial and its derivative over F always have a common root.

   D. Every polynomial over F always has a root in F.

8. Let $\mathbb{F}_p$ denote the finite field with p elements. Then, the map $\phi : \mathbb{F}_p \to \mathbb{F}_p$ given by $\phi(x) = x^p$ is

   A. Only injective but not surjective

   B. Only surjective but not injective

   C. A bijection

   D. Neither surjective nor injective

9. Let $L = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. A primitive element for this extension is:

   A. $\sqrt{2}$

   B. $\sqrt{3}$

   C. $\sqrt{2} + \sqrt{3}$

   D. $\sqrt{6}$

10. If the minimal polynomial of $\sqrt{1+\sqrt{3}}$ is written in the form $X^4 + bX^3 + cX^2 + dX + e$, then the values of c and e are

    A. $c = -2, e = 2.$

    B. $c = 2, e = -2.$

    C. $c = 2, e = 2.$

    D. $c = -2, e = -2.$

Sem-III
PSMT304,PAMT304
PSMT305,PAMT305
GRAPH THEORY
Sample Questions

1. How many edges are there in a simple graph with 10 vertices each of degree six?
   A) 20
   B) 40
   C) 30
   D) 60
2. Which of the following statement is true for a simple graph G with at least two vertices.
   A) G has even number of vertices of odd degree with all distinct degrees of vertices.
   B) G must be two vertices that have the same degree.
   C) It is possible to have G with all distinct degrees of vertices.
   D) G has odd number of vertices of odd degree.
3. Let G be finite simple graph. Then
   A) $\kappa(G) \leq \kappa'(G) \leq \delta(G)$.
   B) $\kappa'(G) \leq \kappa(G) \leq \delta(G)$.
   C) $\delta(G) \leq \kappa(G) \leq \kappa'(G)$.
   D) $\kappa'(G) \leq \delta(G) \leq \kappa(G)$.
4. Let G be a simple connected graph with $|V(G)| \geq 2$. Then, which of the following is true?
   A) G does not have a not cut-vertex.
   B) G has at most one not cut-vertex.
   C) G contains at least 2 vertices which are not cut-vertices.
   D) G contains at most 2 vertices which are not cut-vertices.
5. Let G be a finite simple graph. G is a tree then
   A) G is connected and $|V(G)| = |E(G)| - 1$.
   B) G is connected and $|E(G)| = |V(G)| - 1$.
   C) G is connected graph containing cycle.
   D) G acyclic and $|V(G)| = |E(G)| - 1$.
6. If t is the number of non-isomorphic trees on 4 vertices then
   A) $t = 14$.

B) $t = 12$.

C) $t = 16$.

D) $t = 20$.

7. Let G be a connected graph and $F \subseteq E(G)$. Then F is bond if

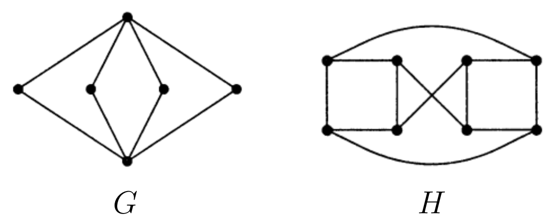
A) $G - F$ has exactly two components.

B) $G - F$ has more than 2 components.

C) $G - F$ has more than 3 components.

D) $G - F$ has more than 3 components.

8. Which of the following statement is true, for graphs G and H given below ?



A) G non-Eulerian and H is Eulerian.

B) G and H both are Eulerian.

C) G and H both are non-Eulerian.

D) G Eulerian and H is non-Eulerian.

9. If $r(m, n)$ denote the Ramsey number then which of the following is true.

A) $r(3, 5) < 14$ and $r(4, 4) < 18$.

B) $r(3, 5) = 14$ and $r(4, 4) = 18$.

C) $r(3, 5) = 14$ and $r(4, 4) < 18$.

D) $r(3, 5) < 14$ and $r(4, 4) = 18$.

10. Maximum number of edges in a non-Hamiltonian graph with 10 vertices is

A) 36

B) 37

C) 38

D) 39

Integral Transforms

1) (1 point) Laplace transform of the function $\sin|t|$ is

A. $\dfrac{1}{1+s^2}$

B. $\dfrac{s}{1+s^2}$

C. $\dfrac{1+e^{-\pi s}}{1-e^{-\pi s}} \cdot \dfrac{1}{1+s^2}$

D. $-\dfrac{1+e^{-\pi s}}{1-e^{-\pi s}} \cdot \dfrac{1}{1+s^2}$

2) (1 point) Which of the following function is of exponential order:

A. $t^3 \sin t$

B. e^{t^2}

C. $\tan t$

D. $\cot t$

3) (1 point) Laplace transform of the function $\sin\sqrt{t}$ is

A. $\dfrac{1}{2s}\sqrt{\dfrac{\pi}{s}} e^{\dfrac{1}{4s}}$, $\mathrm{Re}(s) > 0$

B. $\dfrac{1}{2s}\sqrt{\dfrac{\pi}{2}} e^{-\dfrac{1}{4s}}$, $\mathrm{Re}(s) > 0$

C. $\dfrac{1}{2s}\sqrt{\dfrac{\pi}{s}} e^{-\dfrac{1}{4s}}$, $\mathrm{Re}(s) > 0$

D. $-\dfrac{1}{2s}\sqrt{\dfrac{\pi}{s}} e^{-\dfrac{1}{4s}}$, $\mathrm{Re}(s) > 0$

4) (1 point) If f piecewise continuous on $[0, \infty)$ and is of exponential order c_0, then $\mathcal{L}\left\{\int_0^t f(u)\,du; p\right\}$ is

A. $-\dfrac{F(p)}{p}$

B. $\dfrac{F(p)}{p}$

C. $\dfrac{F(p+1)}{p}$

D. $-\dfrac{F(p+1)}{p}$

5) (2 points) Fourier transform of the signature function is

A. $\sqrt{\frac{2}{\pi}}\frac{i}{s}$

B. $-\sqrt{\frac{2}{\pi}}\frac{i}{s}$

C. $\sqrt{\frac{2}{\pi}}\frac{1}{s}$

D. $\sqrt{\frac{2}{\pi}}\frac{-1}{s}$

6) (2 points) The value of the integral $\int_{-\infty}^{\infty}\frac{1}{(x^2+a^2)^2}\,dx$ is

A. $\frac{\pi}{2a}$

B. $-\frac{\pi}{2a^3}$

C. $-\frac{\pi}{2a}$

D. $\frac{\pi}{2a^3}$

7) (3 points) The Mellin transform of the function $\frac{1}{(x+2)(x+9)}$ is

A. $\frac{-\pi e^{-i\pi s}}{\sin \pi s}\left\{\frac{(-2)^{s-1}}{7}-\frac{(-9)^{s-1}}{7}\right\}$

B. $\frac{\pi e^{-i\pi s}}{\sin \pi s}\left\{\frac{(-2)^{s-1}}{7}-\frac{(-9)^{s-1}}{7}\right\}$

C. $\frac{-\pi e^{-i\pi s}}{\sin \pi s}\left\{\frac{(-2)^{s-1}}{7}+\frac{(-9)^{s-1}}{7}\right\}$

D. $\frac{-\pi e^{-i\pi s}}{\sin \pi s}\left\{\frac{(-2)^{s}}{7}-\frac{(-9)^{s-1}}{7}\right\}$

8) (3 points) Let the Mellin transform of the function $f(x)$ is $F(s)$. Then the Mellin transform of the function $f^{(n)}(x)$, $n \in \mathbb{N}$ is

A. $\frac{\Gamma(s)}{\Gamma(s-n)}F(s-n)$

B. $(-1)^n\frac{\Gamma(s)}{\Gamma(n)}F(s-n)$

C. $\frac{1}{s}F(s-a)$

D. $(-1)^n\frac{\Gamma(s)}{\Gamma(s-n)}F(s-n)$

9) (4 points) The inverse Z transform of the function $F(z) = \dfrac{9z^3}{(3z-1)^2(z-2)}$ is

A. $\dfrac{9}{25}\left\{2^{n+2} - (n+11)\dfrac{1}{3^{n+2}}\right\}$

B. $\dfrac{9}{25}\left\{2^{n+2} + (5n+11)\dfrac{1}{3^{n+2}}\right\}$

C. $\dfrac{9}{25}\left\{2^{n+2} - (5n+11)\dfrac{1}{3^{n+2}}\right\}$

D. $\dfrac{9}{25}\left\{2^{n} - (5n+11)\dfrac{1}{3^{n+2}}\right\}$

10) (4 points) Solution of the difference equation $y_{n+2} - 5y_{n+1} + 6y_n = 2n+1,\ y_0 = 0, y_1 = 1$ is

A. 3^n

B. $(-3)^n$

C. $2 - 2^{n+1} + 2 \cdot 3^n + n$

D. $(-2)^n$

Algebra I

Q1. (1 point) Let $\mathcal{B} = \{(1,0,-1),(1,1,1),(2,2,0)\}$ be a basis for $\mathbb{C}^3$. The dual basis $\mathcal{B}^*$ of $\mathcal{B}$ for $\mathbb{C}^{3^*}$ is given by

1) $f_1(x_1,x_2,x_3) = x_1, f_2(x_1,x_2,x_3) = x_2, f_3(x_1,x_2,x_3) = x_3$
2) $f_1(x_1,x_2,x_3) = -x_1, f_2(x_1,x_2,x_3) = -x_2, f_3(x_1,x_2,x_3) = x_3$
3) $f_1(x_1,x_2,x_3) = x_1 - x_2, f_2(x_1,x_2,x_3) = x_1 - x_2 + x_3, f_3(x_1,x_2,x_3) = -\frac{1}{2}x_1 + x_2 - \frac{1}{2}x_3$
4) $f_1(x_1,x_2,x_3) = 1, f_2(x_1,x_2,x_3) = 0, f_3(x_1,x_2,x_3) = -1$

Q2. (1 point) Let V be the vector space of all polynomial functions over the field of real numbers. Let a and b be fixed real numbers and let f be a linear functional on V defined by $f(p) = \int_a^b p(x)dx$. If D is the differentiation operator on V, then $D^t f$, D^t is transpose of D, is given by

1) 1
2) 0
3) $p(b) - p(a)$
4) $p(a) - p(b)$

Q3. (1 point) Let n be a positive integer and let V be the vector space of all polynomial functions over the field of real numbers which have degree at most n.If D is the differentiation operator on V, then dimension of the null space of D^t, D^t is transpose of D, is given by

1) 0
2) $n-1$
3) 1
4) $n+1$

Q4. (2 points) Let D_1, D_2 be the functions on set of 3×3 matrices over the field of real numbers defined by, for $A = [A_{ij}]_{3\times 3}$, $D_1(A) = A_{11} + A_{22} + A_{33}$ and $D_2(A) = -A_{11}^2 + 3A_{11}A_{22}$. Then

1) D_1 is a 2-linear function
2) D_2 is a 2-linear function
3) both D_1 and D_2 are 2-linear functions
4) both D_1 and D_2 are not 2-linear functions

Q5. (2 points) The determinant of the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is

1) 0

2) $(a-b)(c-a)(c-b)$

3) -1

4) $(b-a)(c-a)(b-c)$

Differential Geometry

1. The curvature of the function $f(x) = x^2 + 2x + 1$ at $x = 0$ is?

   A. 3/2.

   B. 2.

   C. 0.

   D. $|\frac{2}{5^{3/2}}|$.

2. The curvature of the function $f(x) = x^3 - x + 1$ at $x = 1$ is given by?

   A. $|6/5|$.

   B. $|3/5|$.

   C. 0.

   D. $|\frac{6}{5^{3/2}}|$. .

3. The curvature of a function depends directly on leading coefficient when $x = 0$ which of the following could be $f(x)$?

   A. $f(x) = 323x^3 + 4334x + 10102$.

   B. $f(x) = x^5 + 232x^4 + 232x^2 + 12344$.

   C. $f(x) = ax^5 + c$.

   D. $f(x) = 33x^2 + 112345x + 8945$.

4. Consider the curvature of the function $f(x) = e^x$ at $x = 0$. The graph is scaled up by a factor of and the curvature is measured again at $x = 0$. What is the value of the curvature function at $x = 0$ if the scaling factor tends to infinity?

   A. a.

   B. 2.

   C. 1.

   D. 0.

5. Let S_1 be unit sphere in $\mathbb{R}^3$ center at origin and S_2 be another sphere in $\mathbb{R}^3$ of radius 5 center at origin. Let γ_1 and γ_2 be two diametric circles on S_1 and S_2 respectively, and $\kappa_i,\ \tau_i$ are curvatures and torsions of γ_i respectively, for $i = 1,\ 2$. Then

   A. $\kappa_1 = \kappa_2$ and $\tau_1 < \tau_2$,

   B. $\kappa_1 = \kappa_2$ and $\tau_1 > \tau_2$,

   C. $\kappa_1 < \kappa_2$ and $\tau_1 = \tau_2$,

   D. $\kappa_1 > \kappa_2$ and $\tau_1 = \tau_2$,

Algebra I

6. A is a 2×2 unitary matrix. Then eigen value of A are
   - A. 1, -1.
   - B. 1, -i.
   - C. i, -i.
   - D. -1, i.
7. A is 5×5 matrix, all of whose entries are 1, then
   - A. A is not diagonalizable.
   - B. A is idempotent.
   - C. A is nilpotent.
   - D. The minimal polynomial and the characteristics polynomial of A are not equal.
8. A is a 5×5 matrix over $\mathbb{R}$, then $(t^2+1)(t^2+2)$
   - A. is a minimal polynomial.
   - B. is a characteristics polynomial.
   - C. is minimal as well as characteristics polynomial.
   - D. is not minimal as well as characteristics polynomial.
9. M is a 2-square matrix of rank 1, then M is
   - A. diagonalizable and non singular.
   - B. diagonalizable and nilpotent .
   - C. neither diagonalizable nor nilpotent.
   - D. either diagonalizable or nilpotent.
10. Let $p(\mathbb{R})$ be vector space of all polynomials over $\Re$. $T_i : P(\mathbb{R}) \to P(\mathbb{R})$ such that $T_1(f(x)) = \int_0^x f(t)dt$ and $T_2(f(x)) = f^{'}(x)$. Then
    - A. T_1 is 1-1, T_2 is not.
    - B. T_2 is 1-1, T_1 is not.
    - C. T_1 is onto and T_2 is 1-1.
    - D. T_1 and T_2 both are 1-1. .

Algebra II

11. What is the order of the subgroup generated by 20(mod30) in the cyclic group$\mathbb{Z}_{30}$?

    A. 20.

    B. 10.

    C. 6.

    D. 3.

12. An isomorphism $f : \mathbb{R} \longrightarrow \mathbb{R}^+$ from the additive group $\mathbb{R}$ of real numbers onto the multiplicative group $\mathbb{R}^+$ of positive real numbers is defined by

    A. $f(x) = x^2$.

    B. $f(x) = x^3$.

    C. $f(x) = \sin$.

    D. $f(x) = 4^{x+1}$.

13. Consider following statements:

    (I) Every group of order 36 is abelian.

    (II) A group in which every element is of order at most 2 is abelian.

    Pick the correct option:

    A. Only I is correct.

    B. Only II is correct.

    C. Both are correct.

    D. Both are incorrect.

14. Let F and F' be two finite fields with nine and four elements respectively. How many field homomorphisms are there from F to F'?

    A. 3.

    B. 2.

    C. 1.

    D. 0.

15. How many fields are there (up to isomorphism) with exactly 6 elements?

    A. 3.

    B. 2.

    C. 1.

    D. 0 .

Algebra III

16. Let G_1 be semi direct product $\mathbb{Z}_5 \triangleleft \mathbb{Z}_2$ and G_2 be semi direct product $\mathbb{Z}_3 \triangleleft \mathbb{Z}_7$. Then

    A. G_1 is cyclic.

    B. G_2 is abelian.

    C. G_1 is abelian but G_2 is not.

    D. $G_1,\ G_2$ both may not be abelian.

17. Let S_n be a permutation group for $ngeq1$. Consider following statements:

    (I) S_n has composition series for $ngeq7$.

    (II) S_n is solvable for $ngeq7$.

    Pick the correct option:

    A. Statement (I) implies statement (II).

    B. Statement (II) implies statement (I).

    C. Statement (I) is correct but Statement (II) is not.

    D. Statement (I) is correct but Statement (II) is not.

18. Which of the following is a correct statement;

    A. Two finitely generated modules over a PID are isomorphic if and only if they have the same invariant factors (up to units).

    B. $Hom_{\mathbb{Z}}(\mathbb{Q}, \mathbb{Q}) \neq \mathbb{Q}$.

    C. A torsion module is simple if and only if M is cyclic with prime exponent.

    D. A finitely generated rank 1 module is free.

19. Let G and H be solvable groups.

    A. $G \times H$ is solvable.

    B. $G \times H$ is nilpotent.

    C. $G \times H$is abelian.

    D. $G \times H$ has trivial center.

20. Let G be a group of order 10. Then considered G as a $\mathbb{Z}$-module module. Then

    A. G is a torsion $\mathbb{Z}$-module.

    B. G is not a torsion $\mathbb{Z}$ module. .

    C. Every element of G is not of finite order.

    D. G is torsion free.

PSMT/PAMT 302: Mock test Functional Analysis (Sample Questions) **(MCQs)** **Total Number of Questions: 5**

Instructions:

1) All questions carry equal marks.
2) Each question has four options, and only one is correct option.
3) Students has to choose the correct option to answer the question.

Q.1 X is a Baire space if and only if ..... (Complete true statement by choosing correct option from following)

A. given any countable collection $\{U_n\}$ of open sets in X, each of which is dense in X, their intersection $\cap U_n$ is also dense in X.

B. given any countable collection $\{A_n\}$ of closed sets in X, each of which is dense in X, their intersection $\cap U_n$ is also closed dense in X.

C. given any countable collection $\{U_n\}$ of open sets in X, each of which is dense in X, their intersection $\cup U_n$ is also dense in X.

D. None of these.

Q.2 Which of following is true statement.

A. The set of rationals $\mathbb{Q}$ is G_δ set in the reals.

B. If X is compact Hausdroff space then X is Baire space.

C. $\mathbb{Z}_+$ is not Baire space.

D. $\mathbb{R}$ is not a Baire space.

Q.3 Which following is Hilbert space.

A. l^p space $p \neq 2$

B. The space $C[a, b]$ with norm defined as $\|x\| = \max_{t\in J} |x(t)|$ for $x \in C[a, b]$ and $J = [a, b]$.

C. l^p where $p = 2$

D. $L^p[a, b]$ for $p \neq 2$.

Q.4 If $f : X \to \mathbb{C}$ is bounded linear function on normed space X then which of following defines the norm of function f.

A. $\|f\| = \sup_{x\in X} \dfrac{|f(x)|}{\|x\|}$.

B. $\|f\| = \inf_{x\in X} \dfrac{|f(x)|}{\|x\|}$.

C. $\|f\| = \sup_{x\in X,\, x\neq 0} \frac{|f(x)|}{\|x\|}$.

D. $\|f\| = \inf_{x\in X,\, x\neq 0} \frac{|f(x)|}{\|x\|}$.

Q.5 Let p be real valued function defined on vector space X satisfying $p(x+y) \leq p(x)+p(y)$ and $p(\alpha x)\alpha p(x)$ for each $\alpha \geq 0$. Suppose f is linear functional defined on a subspace S and that $f(s) \leq p(s)$ for all $s \in S$. Then there is a linear functional F defined on X such that .... (Complete the statement of Hahn Banach theorem)

A. $F(x) \leq p(x)$ for all x, and $F(s) = f(s)$ for all $s \in S$.

B. $F(x) > p(x)$ for all x, and $F(s) = f(s)$ for all $s \in S$.

C. $F(x) > p(x)$ for all x, and $F(s) \neq f(s)$ for all $s \in S$.

D. $F(x) < p(x)$ for all x, and $F(s) \neq f(s)$ for all $s \in S$.

PSMT/PAMT 102: Analysis-I Mock Test Sample Questions **(MCQs)** **Total Number of Questions: 10**

Instructions:

1) All questions carry equal marks.

2) Each question has four options, and only one is correct option.

3) Students has to choose the correct option to answer the question.

Q.1 Suppose $K \subset Y \subset X$. Then K is compact relative to X if and only if ..... (Complete true statement by choosing correct option from following)

A. K is compact relative to Y

B. $K = \{0\}$.

C. K is closed relative to Y.

D. K is open relative to Y.

Q.2 Let E^0 denotes the set of all interior points of a set E. Then which of following is true.

A. If $E \subset E^0$ then is closed.

B. $\overline{E^0} = E$

C. E^0 is neither open nor closed.

D. E is open if and only if $E^0 = E$.

Q.3 Suppose E is an open set in $\mathbb{R}^n$, f maps E into $\mathbb{R}^m$, and $x \in E$. If there exists a linear transformation A of $\mathbb{R}^n$ into $\mathbb{R}^m$ such that

$$\lim_{h \to 0} \frac{|f(x+h) - f(x) - Ah|}{|h|} = 0,$$

then which of following is correct.

A. f is differentiable at x and $f'(x) = A^T$.

B. f is not differentiable at x.

C. f is differentiable at x and $f'(x) = A$

D. f is differentiable at x and $f'(x) \neq A$.

Q.4 A mapping f of set E into $\mathbb{R}^k$ is said to be bounded ....

A. if there exists real number M such that $|f(x)| \geq M$ for all $x \in E$.

B. if there exists a closed ball $B(0, r) \subset E$ and real number $M > 0$ such that $|f(x)| \geq M$ for all $x \in B(0, r)$.

C. if there exists real number M such that $|f(x)| \leq M$ for all $x \in E$.

D. if there exists a ball $B(x_0, r) \subset E$, such that $|f(x)| \geq \frac{1}{r}$ for all $x \in B(x_0, r)$.

Q.5 If $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by $f(x, y) = xy$, then $D_f(a, b)(x, y) =?$

A. $abxy$

B. $bx + ay$

C. $ax + by$

D. xy

Q.6 Let $f, g : A \to \mathbb{R}$ be integrable. For any partition P of A and sub rectangle S, then which of following is true.

A. $m_S(f) + m_S(g) \leq m_S(f + g)$

B. $m_S(f) + m_S(g) > m_S(f + g)$

C. $M_S(f + g) > M_S(f) + M_S(g)$

D. $m_S(f) + m_S(g) = 0$

Q.7 If $\{A_i\}$ countable collection with $A = \cup_{i=1}^{\infty} A_i$ and each A_i has measure zero. Then which of following is true.

A. measure of A is greater than zero.

B. measure of A is zero.

C. A has content zero.

D. A is compact.

Q.8 Which of following is a statement of Heine Borel theorem.

A. Let F be an open covering of a closed and bounded set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.

B. Let F be an open covering of a closed and bounded set A in $\mathbb{R}^n$. Then any finite sub cover of F does not cover A.

C. Let F be an open covering of a closed set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.

D. Let F be an open covering of set A in $\mathbb{R}^n$. Then a finite sub cover of F also covers A.

Q.9 A function $f : \mathbb{R}^n \to \mathbb{R}^m$ is said to be differentiable at c if ....(choose correct option from following to complete a true statement)

A. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c)+T_c(v)+\|v\|E_c(v)$, where $E_c(v) \to 0$ as $v \to 0$.

B. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c)+T_c(v)+\|v\|E_c(v)$, where $E_c(v) \to \infty$ as $v \to 0$.

C. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c + v) = f(c) + T_c(v) + E_c(v)$, where $E_c(v) \to 0$ as $v \to 0$.

D. There exists function $T_c : \mathbb{R}^n \to \mathbb{R}^m$ such that $f(c+v) = f(c) + T_c(v)$, where $T_c(v) \to 0$ as $v \to 0$.

Q.10 Consider the function $f : (0,1) \to \mathbb{R}$ defined by $f(x) = \dfrac{1}{x}$. Then .....

A. f is not continuous but bounded in $(0,1)$.

B. f is neither continuous nor bounded in $(0,1)$.

C. f is not continuous.

D. f is continuous but not bounded in $(0,1)$.

Answer to this question is (D)

PSMT/PAMT 203: Analysis-II (Mock Test) Sample questions **(MCQs)** **Total Number of Questions: 10**

Instructions:

1) All questions carry equal marks.

2) Each question has four options, and only one is correct option.

3) m denote Lebesgue measure and μ is an arbitrary measure.

Q.1 If$X = \mathbb{R}^n$, and $A = \prod_{i=1}^n (a_i, b_i]$ $a_i, b_i \in \mathbb{R}$ for all $i = 1, 2, \ldots$. Then $m(A) = ...?$

A. 0.
B. $Vol(A) = \prod_{i=1}^n (b_i - a_i)$
C. $\sum_{i=1}^n (b_i - a_i)$
D. 2

Q.2 Let $\mathscr{A}$ be a σ-algebra of subsets of $\mathbb{R}^n$, $E_1, \ldots, E_n$ a finite sequence of disjoint measurable sets. Then which of following is true.

A. $m^*(A \cap [\cup_{i=1}^n E_i]) = \sum_{i=1}^n m^*(A \cap E_i)$
B. $m^*(A \cap [\cup_{i=1}^n E_i]) = \sum_{i=1}^n m^*(A \cup E_i)$
C. $m^*(A \cap [\cup_{i=1}^n E_i]) = \sum_{i=1}^n m^*(A^c \cap E_i)$
D. $m^*(A \cap [\cup_{i=1}^n E_i]) = \sum_{i=1}^n m^*(A \cap E_i^c)$

Q.3 Let X be a measure space and (E_n) be a sequence of measurable sets such that $E_{n+1} \subset E_n$ for each $n \in \mathbb{N}$. Let $\mu(E_1)$ be finite. Then which of following is true.

A. $\mu(\cup_{i=1}^\infty E_i) = \lim_{n\to\infty} \mu(E_n)$.
B. $\mu(\cap_{i=1}^\infty E_i) = \mu(E_1)$.
C. $\mu(\cap_{i=1}^\infty E_i) = \lim_{n\to\infty} \mu(E_n)$.
D. $\mu(\cup_{i=1}^\infty E_i) = \mu(E_n)$.

Q.4 Let A be any set, and $E_1, E_2, \ldots, E_n$ finite collection of disjoint measurable sets. Then ....

A. $\mu^*(A \cup (\cap_{i=1}^n E_i)) = \sum_{i=1}^n \mu^*(A \cap E_i)$.
B. $\mu^*(A \cap (\cap_{i=1}^n E_i)) = \sum_{i=1}^n \mu^*(A \cap E_i)$.
C. $\mu^*(A \cap (\cup_{i=1}^n E_i)) = \sum_{i=1}^n \mu^*(A \cup E_i)$.
D. $\mu^*(A \cap (\cup_{i=1}^n E_i)) = \sum_{i=1}^n \mu^*(A \cap E_i)$.

Q.5 Let X be a measurable space and $E \subseteq X$.

1. E is measurable.

2. Given $\epsilon > 0$, there exists open set $O \supset E$ with $\mu^*(O - E) < \epsilon$.
3. $\mu^*(A) = 0$

Then which of following pair is of equivalent statements.

A. 1 and 2.
B. 2 and 3.
C. 3 and 1.
D. None of these.

Q.6 If f and g are measurable functions which of following function is not measurable.

A. $f + g$
B. fg
C. $|f|$
D. $f\chi_A$ where A is non measurable set.

Q.7 If $f, g : [0, 1] \to \mathbb{R}$ are defined as follows.

$$f(x) = \begin{cases} 0, & 0 < x \leq 1/2 \\ 1, & 1/2 < x \leq 1 \end{cases} \quad g(x) = \begin{cases} 1, & 0 < x < 1/2 \\ 0, & 1/2 \leq x \leq 1. \end{cases}$$

Then which of following is true.

A. $f = g$ a.e.
B. $f \neq g$ a.e.
C. $f \leq g$ a.e.
D. $f \geq g$ a.e.

Q.8 If $f : [0, 1] \to \mathbb{R}$ is defined as follows.

$$f(x) = \begin{cases} 0, & 0 < x \leq 1/2 \\ 1, & 1/2 < x \leq 1 \end{cases}$$

Then $m(\{x; f(x) > 1/2\}) =?$

A. 0
B. 1/2
C. 1
D. $+\infty$

Q.9 If $g : [0, 1] \to \mathbb{R}$ is defined as follows.

$$g(x) = \begin{cases} 0, & 0 < x \leq 1 \\ x - 1, & 1 < x \leq 2 \end{cases}$$

Then $m(\{x; g(x) > 0\}) =?$

A. 0

B. 1/2

C. 1

D. $+\infty$

Q.10 Let $X = [0,1] \times [0,1] \times [0,1] \subseteq \mathbb{R}^3$, $x = (1/2,\ 1/2,\ 1/2)$, then $m(A + x) =?$.

A. 0,

B. 1,

C. 1+1/2

D. $+\infty$

Partial Differential Equations

1. The partial differential equation $y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$ is

   (A) Parabolic

   (B) Hyperbolic

   (C) Elliptic

   (D) Integral Surface.

2. Let $\omega_n = \frac{2\pi^{\frac{n}{2}}}{\Gamma n/2}$ and $B_r(x) = \{y \in I\!R^n / \mid x - y \mid < r\}$. The volume of $B_r(x)$ in $I\!R^n$ is given by

   (A) $\frac{\omega_n}{n}$

   (B) $r\frac{\omega_n}{n}$

   (C) $r^{n-1}\frac{\omega_n}{n}$

   (D) $r^n\frac{\omega_n}{n}$.

3. The general solution of $yuu_x + xuu_y = xy$ is given by

   (A) $x^2 - u^2 = c_1$ where c_1 is a constant

   (B) $y^2 - u^2 = c_2$ where c_2 is a constant

   (C) $h(x^2 - u^2, y^2 - u^2) = 0$ where h is an arbitrary function

   (D) $h(x^2, y^2) = u^2$ where h is an arbitrary function.

4. If $f(x) = \phi(r)$ where $x \in I\!R^n$ and $r = |x|$ then $\Delta f(x)$ equals to

(A) $\phi''(r) + (\frac{n+1}{r})\phi'(r)$

(B) $\phi''(r) + (\frac{n+1}{r^2})\phi'(r)$

(C) $\phi''(r) + (\frac{n-1}{r})\phi'(r)$

(D) $\phi''(r) + (\frac{n-1}{r^2})\phi'(r)$.

5. Suppose u is harmonic on an open set Ω. If $x \in \Omega$ and $r > 0$ is small enough so that $\overline{B_r(x)} \subset \Omega$ then $u(x)$ equals to

(A) $\frac{1}{\omega_n} \int_{S_1(0)} u(x+y)d\sigma(y)$

(B) $\frac{1}{\omega_n} \int_{S_1(0)} u(x-y)d\sigma(y)$

(C) $\frac{1}{\omega_n} \int_{S_1(0)} u(x-ry)d\sigma(y)$

(D) $\frac{1}{\omega_n} \int_{S_1(0)} u(x+ry)d\sigma(y)$

6. The fundamental solution for Laplacian operator Δ is given by

(A) $N(x) = \frac{|x|^{2+n}}{(2+n)\omega_n}$; $(n > 2)$ and $N(x) = \frac{1}{2\pi}\log|x|$; $(n = 2)$

(B) $N(x) = \frac{|x|^{2-n}}{(2-n)\omega_n}$; $(n > 2)$ and $N(x) = \frac{1}{2\pi}\log|x|$; $(n = 2)$

(C) $N(x) = \frac{|x|^{n}}{n\omega_n}$; $(n > 2)$ and $N(x) = \frac{1}{2\pi}\log|x|$; $(n = 2)$

(D) $N(x) = \frac{|x|^{2}}{2\omega_n}$; $(n > 2)$ and $N(x) = \frac{1}{2\pi}\log|x|$; $(n = 2)$

7. The Gaussian kernel $K_t(x)$ defined on $\mathbb{R}^n \times (0, \infty)$ satisfies

(A) $(\Delta_x)K_t(x) = 0$

(B) $(\partial_t - \Delta_x)K_t(x) = 0$

(C) $(\partial_t)K_t(x) = 0$

(D) $(\partial_t^2 - \Delta_x)K_t(x) = 0$.

8. Consider the boundary value problem of heat conduction $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$, $t > 0$ with initial conditions $u(x, 0) = h(x)$ when $-\infty < x < \infty$ and boundary conditions u and $u_x \longrightarrow 0$ as $| x | \longrightarrow \infty$. Let the Fourier transform of $u(x, t)$ and $h(x)$ be denoted by $\mathcal{F}\{u(x, t)\} = U(\alpha, t)$ and $\mathcal{F}\{h(x)\} = H(\alpha)$. The application of Fourier transform converts this boundary value problem in initial value problem as

(A) $U_t + k\alpha^2 U = 0$ with initial condition $U(\alpha, 0) = H(\alpha)$

(B) $U_t - k\alpha^2 U = 0$ with initial condition $U(\alpha, 0) = H(\alpha)$

(C) $U_t + k\alpha U = 0$ with initial condition $U(\alpha, 0) = H(\alpha)$

(D) $U_t - k\alpha U = 0$ with initial condition $U(\alpha, 0) = H(\alpha)$

9. The spherical mean $M_h(x, r)$ satisfies

(A) $\Delta_x M_h(x, r) = [\partial_r^2 + (\frac{n}{r})\partial_r]M_h(x, r)$

(B) $\Delta_x M_h(x, r) = [\partial_r^2 + (\frac{n-1}{r})\partial_r]M_h(x, r)$

(C) $\Delta_x M_h(x, r) = [\partial_r^2 + (\frac{n+1}{r})\partial_r]M_h(x, r)$

(D) $\Delta_x M_h(x, r) = [\partial_r^2 + (\frac{n-2}{r})\partial_r]M_h(x, r)$.

10. Consider one dimensional wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{9}\frac{\partial^2 u}{\partial t^2}$, $-\infty < x < \infty$, $t > 0$ with initial conditions $u(x, 0) = f(x) = 1$ when $| x | \leq 2$ otherwise $u(x, 0) = f(x) = 0$ and $u_t(x, 0) = g(x) = 1$ when $| x | \leq 2$ otherwise $u_t(x, 0) = g(x) = 0$. D'Alembert solution gives the value of $u(0, \frac{1}{6})$ is equals to

(A) $\frac{5}{6}$

(B) $\frac{7}{6}$

(C) $\frac{11}{6}$

(D) $\frac{13}{6}$

Sem-II
PSMT205/PAMT205
PROBABILITY THEORY
(Sample Questions)

1. A mixture of candies contains 6 mints, 4 toffees, and 3 chocolates. If a person makes a random selection of one of these candies, find the probability of getting a toffee or a chocolate.

   A. 9/13

   B. 10/13

   C. 7/13

   D. 13/7

2. Let $\Omega = \{1, 2, 3, 4\}$ and $\mathcal{F}_1 = \{\emptyset, \{1, 2\}, \{3, 4\}\}$,
   $\mathcal{F}_2 = \{\emptyset, \Omega, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{2, 3, 4\}, \{1, 3, 4\}\}$. Then

   A. $\mathcal{F}_2$ is a field but $\mathcal{F}_1$ is not.

   B. $\mathcal{F}_1$ and $\mathcal{F}_2$ both are not fields.

   C. $\mathcal{F}_1$ and $\mathcal{F}_2$ both are fields.

   D. $\mathcal{F}_1$ is a field but $\mathcal{F}_2$ is not.

3. Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

   A. 1/10

   B. 1/19

   C. 1/4

   D. 1/5

4. Karan, Kumar and Kana are participating in the shooting event. The probability that Karan hits a target is 1/4 and the corresponding probabilities for Kumar and Kana are 1/3 an 2/5, respectively. If they all fire together, find the probability that Karan hits the target given that exactly one hit is registered.

   A. 1/8

   B. 3/8

   C. 2/9

   D. 5/9

5. A consulting firm rents cars from three agencies: 30% from agency A, 20% from agency B and 50% from agency C. 15% of the cars from A, 10% of the cars from B and 6% of the cars from C have bad tyres. If a car rented by the firm has bad tyres, find the probability that it came from agency C.

A. 0.2158

B. 0.4158

C. 0.3158

D. 0.5158

6. What is the expectation of a random variable with Bernoulli distribution $B(1, p)$?

   A. $2p$.

   B. p.

   C. 1

   D. $p/2$

7. $f(x) = \begin{cases} \frac{1}{9}x^2, & 0 < x < 3, \\ 0, & elsewhere; \end{cases}$

   is a probability density function of a continuous random variable X. Find $P(X > 1)$.

   A. 25/27

   B. 23/27

   C. 13/27

   D. 26/27

8. A random variable X has the probability density function given by

   $f(x) = \begin{cases} c\sqrt{x}, & 0 < x < 1, \\ 0, & elsewhere. \end{cases}$

   Find the value of the constant c.

   A. 3/2

   B. 1/2

   C. 2/3

   D. 1/3

9. If X and Y are independent random variables and ϕ_X, ϕ_Y denotes characteristic functions of X and Y respectively then,

   A. $\phi_{X+Y}(t) = \phi_X(t) + \phi_Y(t)$.

   B. $\phi_{X+Y}(t) = \phi_{XY}(t)$.

   C. $\phi_{X+Y}(t) = \phi_{X-Y}(t)$.

   D. $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$.

10. A fair coin is tossed independently n times. Let S_n be the number of heads obtained. Use the Chebyshev inequality to find a lower bound of the probability that $\frac{S_n}{n}$ differs from 1/2 by less than 0.1 when n = 100.

A. 1/4

B. 3/4

C. 1/2

D. 1/3

SEM III, Numerical Analysis
Sample MCQ Questions

1. The hexadecimal equivalent of the binary number $(1101001.1110011)_2$ is given by

   A) $(69.E6)_{16}$.

   B) $(59.E6)_{16}$.

   C) $(45.A2)_{16}$.

   D) $(65.A6)_{16}$.

2. The sum of 0.123×10^3 and 0.456×10^2 in 3-digit mantissa form after rounding is given by

   A) 0.1686×10^3.

   B) 0.169×10^3.

   C) 0.168×10^3.

   D) 0.170×10^3.

3. Find a root of the equation $2x = log_{10}x + 7$ between 3 and 4 by Regula Falsi method correct to 3 decimal places.

   A) 3.554

   B) 3.957

   C) 3.245

   D) 3.789

4. Obtain the first iteration in solving the equation $x^3 - 5x + 1 = 0$ by Muller's method.

   A) 0.191857.

   B) 0.20.

   C) 0.22567.

   D) 0.21574.

5. Using Bairstow's method obtain the quadratic factor of the equation $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$, with $(p, q) = (2, 2)$.

   A) $p = 2.1436, q = 2.1876$

   B) $p = 1.9512, q = 1.9672$

   C) $p = 1.9413, q = 1.9538$

   D) $p = 2.1654, q = 2.1982$

6. Solve the system of equations using Cholesky's method.

$$\begin{aligned} x + 2y + 3z &= 5, \\ 2x + 8y + 22z &= 6, \\ 3x + 22y + 82z &= -10. \end{aligned}$$

A) $(x, y, z) = (1, 2, 3)$

B) $(x, y, z) = (2, 3, 1)$

C) $(x, y, z) = (3, 1.2)$

D) $(x, y, z) = (2, 3, -1)$

7. Let P_k be the sum of the modulii of the elements along the k^{th} row excluding the diagonal element a_{kk}. Then by Brauer's theorem every eigenvalue λ of A satisfies

   A) $|\lambda| \leq \max_i \left[\sum_{j=1}^{n} |a_{ij}|\right]$

   B) $|\lambda - a_{kk}| \leq P_k$

   C) $|\lambda| = \max_i \left[\sum_{j=1}^{n} |a_{ij}|\right]$

   D) $|\lambda - a_{kk}| = P_k$

8. The Jacobi iteration scheme $\mathbf{x^{(k+1)}} = H\mathbf{x^{(k)}} + \mathbf{c},\ k = 0, 1, 2, \ldots$ for finding the solution of a system of equations $A\mathbf{X} = \mathbf{b}$ converges if

   A) $||A|| < 1$

   B) $||A|| \geq 1$

   C) $||H|| < 1$

   D) $||H|| \geq 1$

9. Using composite Simpson's rule evaluate $\displaystyle\int_0^1 \frac{dx}{1+x}$ with 4 equal subintervals.

   A) 0.694444

   B) 0.708333

   C) 0.693254

   D) 0.693155

10. The wieght function $w(x)$ in Gauss Laguerre integration formula
    $\displaystyle\int_a^b w(x)f(x)dx = \sum_{k=0}^{n} \lambda_k f_k$ is

    A) 1

    B) $\dfrac{1}{\sqrt{1-x^2}}$

    C) e^{-x^2}

    D) e^{-x}

****

1. Let v be a vector in a Hilbert space $(\mathcal{H}, < \cdot, \cdot >)$. If $< x, v >= 0$ for all $x \in \mathcal{H}$ then
(A) $||v|| = 0$
(B) $||v|| \neq 0$
(C) x and v are linearly independent
(D) x and v are linearly dependent

2. Let X denote a Banach space and X^* denote its dual. Then
(A) always $(X^*)^* = X^*$
(B) always $(X^*)^* = X$
(C) always $X \subseteq (X^*)^*$
(D) always $(X^*)^* \subseteq X$

3. An orthonormal set $O = \{e_i\}$ in a Hilbert space $(\mathcal{H}, < \cdot, \cdot >)$ is complete if and only if
(A) every Cauchy sequence in the set O is convergent.
(B) $< x, e_i >= 0$ for all $x \in \mathcal{H} \Rightarrow x = 0$;
(C) $< x, e_i >= 0$ for all $x \in O \Rightarrow e_k = 0$ for some k;
(D) O is countable.

4. Which of the following is true?
(A) A complete metric space is always a Hilbert space;
(B) A complete metric space is always a Banach space;
(C) A complete metric space is always a Normed linear space;
(D) A complete metric space is always a Baire space

5. For a normed linear space X, the fact that every closed and bounded set is compact is equivalent to
(A) X is complete
(B) X is a bounded set
(C) the set $\{x \in X : ||x - a|| \leq R\}$ is compact for any $a \in X$ and $R > 0$
(D) X is closed.

1. The equation $a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u)$ where a, b depends on u is called ———

   A. homogeneous

   B. semilinear

   C. linear

   D. quasilinear

2. Consider the equation $yu_x - xu_y = 0$ which gives characteristic curves

   A. $x^2 = y$

   B. $y^2 = constant$

   C. $y = x$

   D. $x^2 + y^2 = constant$

3. The characteristic equations of $u_t + 2xtu_x = e^t$ are ———

   A. $\frac{dt}{ds} = 2x, \quad \frac{dx}{ds} = 2t, \quad \frac{du}{ds} = e^t$

   B. $\frac{dt}{ds} = 1, \quad \frac{dx}{ds} = 2xt, \quad \frac{du}{ds} = e^t$

   C. $\frac{dt}{ds} = 2, \quad \frac{dx}{ds} = xt, \quad \frac{du}{ds} = e^t$

   D. $\frac{dt}{ds} = 1, \quad \frac{dx}{ds} =, \quad \frac{du}{ds} = e^t$

4. The general solution of $u_x + u_y - u = 0$ is ———

   A. $u(x,y) = G(y-x)e^x$

   B. $u(x,y) = G(y+x)e^x$

   C. $u(x,y) = G(yx)e^x$

   D. $u(x,y) = G(y^2 - x)e^x$

5. If $u_1(x,y) = \cos(\beta x - \alpha y), u_2(x,y) = (\beta - \alpha y)^3$ are solutions of the PDE $\alpha u_x + \beta u_y = 0$, where α, β are constants then ——— is also a solution of this PDE.

   A. $3u_1 - 5u_2$

   B. $3u_1 + 5u_2$

   C. $3u_1 - 3u_2$

   D. $3u_1 + 3u_2$

6. The general solution of the linear partial differential equation $a(x,y,u)u_x + b(x,y,u)u_y = c(x,y,u)$ is

   A. $F(\phi, \psi) = 1$

   B. $F(\phi, \psi) = 0$

   C. $F(\phi, \psi) = -1$

D. $F(\phi, \psi) = 2$

7. Characteristics for the equation $(y^2z)z_x + (zx)z_y = y^2$ are ———
   A. $\frac{dx}{y^2z} = \frac{dy}{y^2z} = \frac{dz}{y^2z}$
   B. $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{xz}$
   C. $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{xz}$
   D. $\frac{dx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$

8. Let P, Q, R be C^1 functions of x, y and z. The general solution of $P(x, y, z)p + Q(x, y, z)q = R(x, y, z)$ is ... where F is an arbitrary smooth function of u and v and $u(x, y, z) = C_1$ and $v(x, y, z) = C_2$
   A. $F(u) = 0$
   B. $F(v) = 0$
   C. $F(u, v) = 0$
   D. $F(\frac{u}{v}) = 0$

9. The general solution of $x^2p + y^2q = (x + y)z$ is ———
   A. $F(\frac{1}{x} - \frac{1}{y}, \frac{z}{x-y})$
   B. $F(\frac{1}{x}, \frac{1}{y})$
   C. $F(\frac{1}{x}, \frac{1}{x-y})$
   D. $F(\frac{1}{y}, \frac{1}{x-y})$

10. Consider $y'' + P(x)y' + Q(x)y = 0$. If both $P(x)$ and $Q(x)$ are analytic at p then p is called ———
    A. singular point
    B. ordinary point
    C. regular singular point
    D. both regular and ordinary point

1. If every two elements of a poset are comparable then the poset is called

   A. sub ordered poset

   B. totally ordered poset

   C. sub lattice

   D. semigroup

2. The composition of function is associative but not ———

   A. identity

   B. idempotent

   C. distributive

   D. commutative

3. The set of all real numbers in the interval $(0, 1)$ is —–.

   A. countable

   B. uncountable

   C. finite

   D. infinite

4. The set of rational numbers $\mathbb{Q}$ is ———

   A. countable

   B. finie

   C. uncountable

   D. denumerable

5. A finite set is —– to any of its proper subset.

   A. not equivalent

   B. equivalent

   C. equal

   D. not equal

6. Let $X = \{a, b, c\}$. Then $|\mathcal{P}(X)| = ---$

   A. 3

   B. 5

   C. 8

   D. 24

7. There exists no —– from a set to its power set

A. surjection

B. injection

C. bijection

D. homomorphism

8. Let S be a partially ordered set. If every totally ordered subset of S has an upper bound, then S contains a —— element.

   A. maximal

   B. minimal

   C. lub

   D. glb

9. Self-complemented, distributive lattice is called ——

   A. Boolean algebra

   B. Modular lattice

   C. complete lattice

   D. bounded lattice

10. If S is uncountable and T is countable then $S \setminus T$ is ———.

    A. finite

    B. infinite

    C. countable

    D. uncountable

Topology (Sem III)

1. Which of the following T is a topology on $X = \{a, b, c\}$?

   A. $T = \{X, \phi, \{a\}, \{c\}\}$

   B. $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}\}$

   C. $T = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

   D. $T = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$

2. Let $X = \{a, b, c, d\}$ and let $T = \{\phi, X, \{a\}, \{c, d\}, \{a, c, d\}\}$ be a topology on X. If $E = \{a, d\}$ is a subset of X then subspace topology T_E on E is ...

   A. $T_E = \{\phi, E, \{a\}, \{c\}\}$.

   B. $T_E = \{\phi, X, \{a\}\}$.

   C. $T_E = \{\phi, X, \{a\}, \{a, b\}\}$.

   D. $T_E = \{\phi, E, \{a\}, \{d\}\}$.

3. Let $T = \{\phi, X, \{1\}, \{2, 3\}\}$ be a topology on $X = \{1, 2, 3\}$ and let $T' = \{\phi, Y, \{w\}, \{u, v\}\}$ be a topology on $Y = \{u, v, w\}$. A function $f : X \to Y$ is defined by $f(1) = w$, $f(2) = u$, $f(3) = v$. Then ....

   A. f is a continuous function.

   B. f is not a onto function.

   C. f is not a one-one function.

   D. f is not open.

4. If the sets A and B form a separation of X and if Y is a connected subspace of X then ....

   A. either $Y \subset A$ or $Y \subset B$.

   B. either $A \subset Y$ or $B \subset Y$.

   C. either $\overline{A} \subset Y$ or $\overline{B} \subset Y$.

   D. either $Y \not\subseteq A$ or $Y \not\subseteq B$.

5. A continuous image of a connected space is .....

   A. totally disconnected.

   B. disconnected.

   C. connected.

   D. both totally disconnected and disconnected.

6. A topological space (X, T) is separable if there exists a countable subset A of X such that ...

   A. $A = X$

   B. $A \neq X$

C. $\overline{A} = X$

D. $\overline{A} \neq X$

7. Every closed subspace of a compact space is ....

   A. compact.

   B. not compact.

   C. neither open nor closed.

   D. unbounded.

8. Let $T = \{\phi, X, \{1\}, \{2\}, \{1,2\}\}$ be a topology on a set $X = \{1,2,3\}$. Then open cover of X is ....

   A. $C = \{\{1\}, \{2,3\}\}$.

   B. $C = X$.

   C. $C = \{\{1\}, \{2\}, \{3\}\}$.

   D. $C = \{\{2\}, \{2,3\}\}$.

9. Under the metric $d(a,b) = |a - b|, a, b \in \mathbb{R}$, the real line $\mathbb{R}$ is ....

   A. complete but not totally bounded.

   B. not complete but totally bounded.

   C. neither complete nor totally bounded.

   D. totally bounded.

10. A metric space (X, d) is compact iff it is ....

    A. complete and not totally bounded.

    B. not complete but totally bounded.

    C. neither complete nor totally bounded.

    D. complete and totally bounded.